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Linearizing $N = 1$ nonlinear supersymmetry with higher derivative terms of a Nambu-Goldstone fermion

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Abstract

We investigate for $N = 1$ supersymmetry (SUSY) the relation between a scalar supermultiplet of linear SUSY and a nonlinear (NL) SUSY model including apparently pathological higher derivaive terms of a Nambu-Goldstone (N-G) fermion besides the Volkov-Akulov (V-A) action. SUSY invariant relations with higher derivative terms of the N-G fermion, which connect the linear and NL SUSY models, are constructed at leading orders by heuristic arguments. We discuss a higher derivative action of the N-G fermion in the NL SUSY model, which apparently includes a (Weyl) ghost field. By using this relation, we also explicitly prove an equivalence between the standard NL SUSY V-A model and our NL SUSY model with the pathological higher derivatives as an example with respect to the universality of NL SUSY actions with the N-G fermion.

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There are two different realizations of supersymmetry (SUSY), the linear realization [1], and the nonlinear realization [2] which characterize Nambu-Goldstone (N-G) fermions [3] indicating the spontaneously SUSY breaking (SSB) [4, 5]. The relation between linear and NL SUSY models was investigated by many authors [6]-[10] as an analogy with internal symmetries. Indeed, for $N = 1$ SUSY it is well-known [6]-[8] that the Volkov-Akulov (V-A) model [2] of NL SUSY is algebraicly equivalent to a scalar supermultiplet of linear SUSY [1]. The relationship between the V-A model and a (axial vector) gauge supermultiplet with the Fayet-Iliopoulos (F-I) D term indicating SSB was also studied in [6, 9]. For $N = 2$ SUSY we have proved by heuristic arguments [10] that the V-A model is equivalent to a (vector) gauge supermultiplet with general F-I D terms, which has a SU(2) ($\times$ U(1)) symmetry.

On the other hand, a (NL SUSY invariant) higher derivative action of the N-G fermion besides the V-A action was explicitly shown in the superspace formalism (the construction of V-A superfield) [11], while in the model towards the SUSY composite unified model which is called the superon-graviton model (SGM) [12] possessing a new NL SUSY in curved spacetime. In particular, in the context of SGM it is inevitable to linearize NL SUSY and to obtain a linear SUSY invariant action which corresponds to the (NL SUSY invariant) SGM action including the higher derivative terms of the N-G fermion in order to derive the low energy physics of SGM. From these viewpoint, it is useful to recognize how a supermultiplet of linear SUSY is expressed in terms of a (NL SUSY invariant) higher derivative action of the N-G fermion in addition to the V-A action. Also, it is important to know the relation of the actions with the N-G fermion and the standard V-A action, i.e., the universality of actions with the N-G fermion as discussed already in Ref.[13].

In this letter, we focus on $N = 1$ SUSY for simplicity and investigate a relation between the scalar supermultiplet action of linear SUSY [1] and a NL SUSY model including apparently pathological higher derivative terms of the N-G fermion besides the V-A action. We show by heuristic arguments that SUSY invariant relations with higher derivative terms of the N-G fermion, which connect the linear and NL SUSY models, are constructed at leading orders starting from an ansatz with a higher (first-order) derivative term of the N-G fermion as given below in Eq.(3). We also briefly discuss a higher derivative action of the N-G fermion in the NL SUSY model. Our model includes higher derivative terms of the N-G fermion which apparently describe a (Weyl) ghost field. By using this relation, we also explicitly prove an equivalence between the standard NL SUSY V-A model and our NL SUSY model with the pathological higher derivatives of the N-G fermion. This is a different example from the arguments of [13] with respect to the universality of NL SUSY actions with the N-G fermion.

Let us denote the component fields of a $N = 1$ scalar supermultiplet [1] as

(A, B, λ, F, G) ‡, in which A and B are two physical scalar fields, λ is a Majorana spinor, and F and G means two auxiliary scalar fields. The linear SUSY transformations of these component fields generated by a constant (Majorana) spinor parametar ζ are written by

$$\begin{aligned}
&\delta_Q A = \bar{\zeta}\lambda, \\
&\delta_Q B = i\bar{\zeta}\gamma_5\lambda, \\
&\delta_Q \lambda = \{(F + i\gamma_5 G) - i\partial\!\!\!/(A + i\gamma_5 B)\}\zeta, \\
&\delta_Q F = -i\bar{\zeta}\partial\!\!\!/\lambda, \\
&\delta_Q G = \bar{\zeta}\gamma_5\partial\!\!\!/\lambda.
\end{aligned} \tag{1}$$

These transformations satisfy a closed off-shell commutator algebra, $[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v)$, where δ_P is a translation with a parameter $v^a = 2i\bar{\zeta}_1\gamma^a\zeta_2$.

On the other hand, for the $N = 1$ V-A model [2] we have a NL SUSY transformation law of a (Majorana) N-G fermion ψ generated by ζ,

$$\delta_Q\psi = \frac{1}{\kappa}\zeta - i\kappa(\bar{\zeta}\gamma^a\psi)\partial_a\psi, \tag{2}$$

where κ is a constant whose dimension is $(\text{mass})^{-2}$, and Eq.(2) also satisfies the off-shell commutator algebra, $[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v)$.

As for the method of constructing SUSY invariant relations between the component fields of the $N = 1$ scalar supermultiplet and the N-G fermion field ψ, there is a heuristic method [7] starting from an ansatz, $\lambda = \psi + \mathcal{O}(\kappa^2)$, and obtaining higher order terms such that the linear SUSY transformations (1) are reproduced by using the NL SUSY transformation (2). In this letter, following this method and starting from the following ansatz with a higher (first-order) derivative term of ψ,

$$\lambda \ = \ \psi \ + \ i\kappa^{\frac{1}{2}} \ \partial\!\!\!/\psi + \ \mathcal{O}(\kappa^2) \ + \ \mathcal{O}(\kappa^{5/2}), \tag{3}$$

we construct the SUSY invariant relations between the component fields of the $N = 1$ scalar supermultiplet and the N-G fermion field ψ at leading orders.

Indeed, after some calculations we obtain the relations between the fields of $N = 1$ scalar supermultiplet and the N-G fermion ψ as

$$A = \ \frac{1}{2}\kappa \ \bar{\psi}\psi - \frac{i}{4}\kappa^3 \ \{(\bar{\psi}\partial\!\!\!/\psi)\bar{\psi}\psi - (\bar{\psi}\gamma_5\partial\!\!\!/\psi)\bar{\psi}\gamma_5\psi\} + \mathcal{O}(\kappa^5)$$

‡In this letter Minkowski spacetime indices are denoted by $a, b, ... = 0, 1, 2, 3$, and we use the Minkowski spacetime metric $\frac{1}{2}\{\gamma^a, \gamma^b\} = \eta^{ab} = (+, -, -, -)$ and $\sigma^{ab} = \frac{i}{4}[\gamma^a, \gamma^b]$.

$$
\begin{aligned}
&+i\kappa^{\frac{3}{2}}\ \bar{\psi}\partial\!\!\!/\psi+\kappa^{\frac{7}{2}}\ \Big[\ \frac{1}{4}\partial_a\{(\bar{\psi}\partial^a\psi)\bar{\psi}\psi-(\bar{\psi}\gamma_5\partial^a\psi)\bar{\psi}\gamma_5\psi\}\\
&-\frac{i}{2}\{(\partial_a\bar{\psi}\sigma^{ab}\partial_b\psi)\bar{\psi}\psi-(\partial_a\bar{\psi}\gamma_5\sigma^{ab}\partial_b\psi)\bar{\psi}\gamma_5\psi\}\\
&+\frac{i}{2}\epsilon^{abcd}(\bar{\psi}\gamma_c\partial_a\psi)\bar{\psi}\gamma_5\gamma_d\partial_b\psi\\
&+\frac{1}{2}\{(\bar{\psi}\partial\!\!\!/\psi)\bar{\psi}\partial\!\!\!/\psi-(\bar{\psi}\gamma^a\partial_b\psi)\bar{\psi}\gamma^b\partial_a\psi\}\ \Big]+\mathcal{O}(\kappa^{\frac{11}{2}}),
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
B=\ &\frac{i}{2}\kappa\ \bar{\psi}\gamma_5\psi+\frac{1}{4}\kappa^3\ \{(\bar{\psi}\partial\!\!\!/\psi)\bar{\psi}\gamma_5\psi-(\bar{\psi}\gamma_5\partial\!\!\!/\psi)\bar{\psi}\psi\}+\mathcal{O}(\kappa^5)\\
&-\kappa^{\frac{3}{2}}\ \bar{\psi}\gamma_5\partial\!\!\!/\psi+i\kappa^{\frac{7}{2}}\ \Big[\ \frac{1}{4}\partial_a\{(\bar{\psi}\partial^a\psi)\bar{\psi}\gamma_5\psi-(\bar{\psi}\gamma_5\partial^a\psi)\bar{\psi}\psi\}\\
&-\frac{i}{2}\{(\partial_a\bar{\psi}\sigma^{ab}\partial_b\psi)\bar{\psi}\gamma_5\psi-(\partial_a\bar{\psi}\gamma_5\sigma^{ab}\partial_b\psi)\bar{\psi}\psi\}\\
&+\frac{i}{2}\epsilon^{abcd}(\bar{\psi}\gamma_5\gamma_c\partial_a\psi)\bar{\psi}\gamma_5\gamma_d\partial_b\psi\\
&+\frac{1}{2}\{(\bar{\psi}\partial\!\!\!/\psi)\bar{\psi}\gamma_5\partial\!\!\!/\psi-(\bar{\psi}\gamma^a\partial_b\psi)\bar{\psi}\gamma_5\gamma^b\partial_a\psi\}\ \Big]+\mathcal{O}(\kappa^{\frac{11}{2}}),
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
\lambda=\ &\psi+\frac{i}{2}\kappa^2\ \{-(\bar{\psi}\partial\!\!\!/\psi)\psi+(\bar{\psi}\gamma_5\partial\!\!\!/\psi)\gamma_5\psi-(\bar{\psi}\partial_a\psi)\gamma^a\psi-(\bar{\psi}\gamma_5\partial_a\psi)\gamma_5\gamma^a\psi\}\\
&+\mathcal{O}(\kappa^4)\\
&+i\kappa^{\frac{1}{2}}\ \partial\!\!\!/\psi\\
&+\kappa^{\frac{5}{2}}\ \Big[\ \frac{1}{2}\partial_a\{(\bar{\psi}\partial^a\psi)\psi-(\bar{\psi}\gamma_5\partial^a\psi)\gamma_5\psi+(\bar{\psi}\partial\!\!\!/\psi)\gamma^a\psi+(\bar{\psi}\gamma_5\partial\!\!\!/\psi)\gamma_5\gamma^a\psi\}\\
&-i\{(\partial_a\bar{\psi}\sigma^{ab}\partial_b\psi)\psi+(\partial_a\bar{\psi}\gamma_5\sigma^{ab}\partial_b\psi)\gamma_5\psi\}\\
&+\frac{i}{2}\epsilon^{abcd}\{(\bar{\psi}\gamma_5\gamma_c\partial_a\psi)\gamma_d\partial_b\psi+(\bar{\psi}\gamma_c\partial_a\psi)\gamma_5\gamma_d\partial_b\psi\}\\
&+i\{(\bar{\psi}\partial_a\psi)\sigma^{ab}\partial_b\psi-(\bar{\psi}\gamma_5\partial_a\psi)\gamma_5\sigma^{ab}\partial_b\psi\}\ \Big]+\mathcal{O}(\kappa^{\frac{9}{2}}),
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
F=\ &\left(\frac{1}{\kappa}-i\kappa\bar{\psi}\partial\!\!\!/\psi\right)+\frac{1}{2}\kappa^3\ \Big[\ i\{(\partial_a\bar{\psi}\sigma^{ab}\partial_b\psi)\bar{\psi}\psi-(\partial_a\bar{\psi}\gamma_5\sigma^{ab}\partial_b\psi)\bar{\psi}\gamma_5\psi\}\\
&-(\partial_a\bar{\psi}\gamma_5\partial\!\!\!/\psi)\bar{\psi}\gamma_5\gamma^a\psi+(\bar{\psi}\gamma^b\partial_a\psi)\bar{\psi}\gamma^a\partial_b\psi-(\bar{\psi}\gamma_5\gamma^b\partial_a\psi)\bar{\psi}\gamma_5\gamma^a\partial_b\psi\\
&-\frac{1}{4}\{(\bar{\psi}\psi)\Box(\bar{\psi}\psi)-(\bar{\psi}\gamma_5\psi)\Box(\bar{\psi}\gamma_5\psi)\}\ \Big]+\mathcal{O}(\kappa^5)\\
&+\kappa^{\frac{3}{2}}\ \partial_a(\bar{\psi}\gamma^a\partial\!\!\!/\psi)-i\kappa^{\frac{7}{2}}\ \Big[\ \frac{1}{4}\{\Box(\bar{\psi}\partial\!\!\!/\psi)\bar{\psi}\psi-\Box(\bar{\psi}\gamma_5\partial\!\!\!/\psi)\bar{\psi}\gamma_5\psi\}
\end{aligned}
$$

$$
\begin{aligned}
&+\left(\frac{1}{4}\partial_a\bar{\psi}\gamma_5\gamma^a\gamma_b\Box\psi+\frac{1}{2}\partial_a\partial_c\bar{\psi}\gamma_5\gamma^c\gamma_b\partial^a\psi-2i\partial_a\partial_b\bar{\psi}\gamma_5\sigma^{ac}\partial_c\psi\right)\bar{\psi}\gamma_5\gamma^b\psi\\
&+(\partial_a\bar{\psi}\partial\!\!\!/\psi)\bar{\psi}\partial^a\psi-(\partial_a\bar{\psi}\gamma_5\partial\!\!\!/\psi)\bar{\psi}\gamma_5\partial^a\psi-(\partial_a\bar{\psi}\partial_b\psi)\bar{\psi}\gamma^a\partial^b\psi\\
&+(\partial_a\bar{\psi}\gamma_5\partial_b\psi)\bar{\psi}\gamma_5\gamma^a\partial^b\psi-i\{(\partial_a\bar{\psi}\sigma^{ab}\partial_b\psi)\bar{\psi}\partial\!\!\!/\psi+(\partial_a\bar{\psi}\gamma_5\sigma^{ab}\partial_b\psi)\bar{\psi}\gamma_5\partial\!\!\!/\psi\}\\
&+\frac{i}{2}\epsilon^{abcd}\{(\partial_e\bar{\psi}\gamma_5\gamma_c\partial_a\psi)\bar{\psi}\gamma^e\gamma_d\partial_b\psi-(\partial_e\bar{\psi}\gamma_c\partial_a\psi)\bar{\psi}\gamma_5\gamma^e\gamma_d\partial_b\psi\}\\
&+4i(\partial_a\bar{\psi}\sigma^{bc}\partial_c\psi)\bar{\psi}\gamma^a\partial_b\psi\ \Bigg]+\mathcal{O}(\kappa^{\frac{11}{2}}),
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
G=\ &\kappa\ \bar{\psi}\gamma_5\partial\!\!\!/\psi-\frac{i}{2}\kappa^3\ \Big[\ i\{(\partial_a\bar{\psi}\gamma_5\sigma^{ab}\partial_b\psi)\bar{\psi}\psi-(\partial_a\bar{\psi}\sigma^{ab}\partial_b\psi)\bar{\psi}\gamma_5\psi\}\\
&+(\partial_a\bar{\psi}\partial\!\!\!/\psi)\bar{\psi}\gamma_5\gamma^a\psi+\frac{1}{4}\{(\bar{\psi}\gamma_5\psi)\Box(\bar{\psi}\psi)-(\bar{\psi}\psi)\Box(\bar{\psi}\gamma_5\psi)\}\ \Big]+\mathcal{O}(\kappa^5)\\
&+i\kappa^{\frac{3}{2}}\ \partial_a(\bar{\psi}\gamma_5\gamma^a\partial\!\!\!/\psi)+\kappa^{\frac{7}{2}}\ \Big[\ \frac{1}{4}\{\Box(\bar{\psi}\partial\!\!\!/\psi)\bar{\psi}\gamma_5\psi-\Box(\bar{\psi}\gamma_5\partial\!\!\!/\psi)\bar{\psi}\psi\}\\
&+\left(\frac{1}{4}\partial_a\bar{\psi}\gamma^a\gamma_b\Box\psi+\frac{1}{2}\partial_a\partial_c\bar{\psi}\gamma^c\gamma_b\partial^a\psi-2i\partial_a\partial_b\bar{\psi}\sigma^{ac}\partial_c\psi\right)\bar{\psi}\gamma_5\gamma^b\psi\\
&+(\partial_a\bar{\psi}\partial\!\!\!/\psi)\bar{\psi}\gamma_5\partial^a\psi-(\partial_a\bar{\psi}\gamma_5\partial\!\!\!/\psi)\bar{\psi}\partial^a\psi+(\partial_a\bar{\psi}\partial_b\psi)\bar{\psi}\gamma_5\gamma^a\partial^b\psi\\
&-(\partial_a\bar{\psi}\gamma_5\partial_b\psi)\bar{\psi}\gamma^a\partial^b\psi-i\{(\partial_a\bar{\psi}\sigma^{ab}\partial_b\psi)\bar{\psi}\gamma_5\partial\!\!\!/\psi+(\partial_a\bar{\psi}\gamma_5\sigma^{ab}\partial_b\psi)\bar{\psi}\partial\!\!\!/\psi\}\\
&+\frac{i}{2}\epsilon^{abcd}\{(\partial_e\bar{\psi}\gamma_5\gamma_c\partial_a\psi)\bar{\psi}\gamma_5\gamma^e\gamma_d\partial_b\psi-(\partial_e\bar{\psi}\gamma_c\partial_a\psi)\bar{\psi}\gamma^e\gamma_d\partial_b\psi\}\\
&+4i(\partial_a\bar{\psi}\gamma_5\sigma^{bc}\partial_c\psi)\bar{\psi}\gamma^a\partial_b\psi\ \Bigg]+\mathcal{O}(\kappa^{\frac{11}{2}}).
\end{aligned}
\tag{8}
$$

It is straightforward but lengthy to prove that the linear SUSY transformations (1) are reproduced by using the NL SUSY transformation (2). The SUSY invariant relations at $\mathcal{O}(\kappa^{2m})$ $(m=0,1,2,\cdots)$ or $\mathcal{O}(\kappa^{2m+1})$ in Eqs. from (4) to (8) are those obtained in [6]-[8]. And also the relation (7) at $\mathcal{O}(\kappa^{2m+1})$ for the auxiliary field F has the form which is proportional to a determinant $|w|=\det(w^a{}_b)$ in $N=1$ V-A model [2] with $w^a{}_b$ being defined by

$$
w^a{}_b=\delta^a{}_b+t^a{}_b,\quad t^a{}_b=-i\kappa^2\bar{\psi}\gamma^a\partial_b\psi,
\tag{9}
$$

plus total derivative terms [7]; namely, Eq.(7) becomes $F=(1/\kappa)|w|+[\text{ tot. der. }]$ which shows that $1/\kappa$ corresponds to the vacuum expectation value of the auxiliary field F. The SUSY invariant relations at $\mathcal{O}(\kappa^{2m+\frac{1}{2}})$ or $\mathcal{O}(\kappa^{2m+\frac{3}{2}})$ are the new higher derivative terms of the N-G fermion.

The derivation of the above SUSY invariant relations from (4) to (8) does not depend on the form of the action for the linear and NL SUSY models. We now

consider a free action which is invariant under Eq.(1)

$$S_{\rm lin} = \int d^4x \left[\frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a B)^2 + \frac{i}{2}\bar{\lambda}\partial\!\!\!/ \lambda + \frac{1}{2}(F^2 + G^2) - \frac{1}{\kappa}F \right]. \tag{10}$$

The last term proportional to κ^{-1} is an analog of the Fayet-Iliopoulos D term in the $N = 1$ gauge supermultiplet [4]. The field equations for the auxiliary field F is $F = 1/\kappa$ indicating a spontaneous SUSY breaking. As already shown in [6]-[8], substituting the terms at $\mathcal{O}(\kappa^{2m})$ and $\mathcal{O}(\kappa^{2m+1})$ of Eqs. from (4) to (8) into the linear action $S_{\rm lin}$ of Eq.(10) gives the V-A action $S_{\rm VA}$,

$$\begin{aligned} S_{\rm VA} = & -\frac{1}{2\kappa^2}\int d^4x \ |w| \\ = & -\frac{1}{2\kappa^2}\int d^4x \Big[1 + t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) \\ & -\frac{1}{6}\epsilon_{abcd}\epsilon^{efgd} t^a{}_e t^b{}_f t^c{}_g - \frac{1}{4!}\epsilon_{abcd}\epsilon^{efgh} t^a{}_e t^b{}_f t^c{}_g t^d{}_h \Big], \end{aligned} \tag{11}$$

which is invariant under the NL SUSY transformation (2).

When we substitute the terms at both ($\mathcal{O}(\kappa^{2m})$, $\mathcal{O}(\kappa^{2m+1})$) and ($\mathcal{O}(\kappa^{2m+\frac{1}{2}})$, $\mathcal{O}(\kappa^{2m+\frac{3}{2}})$) of Eqs. from (4) to (8) into the linear action $S_{\rm lin}$ of Eq.(10), $S_{\rm lin}$ leads a higher derivative action $S_{\rm higher\ der}$ with respect to ψ in addition to the V-A action $S_{\rm VA}$ of Eq.(11); namely, $S_{\rm lin} = S_{\rm VA} + S_{\rm higher\ der}$, and $S_{\rm higher\ der}$ at $\mathcal{O}(\psi^2)$ (i.e. up to $\mathcal{O}(\kappa)$) is given for example as

$$S_{\rm higher\ der}[\mathcal{O}(\psi^2)] = \int d^4x \left[\kappa^{1/2}\partial_a\bar{\psi}\partial^a\psi + \frac{i}{2}\kappa\partial_a\bar{\psi}\gamma^a\Box\psi \right] \tag{12}$$

except for total derivative terms. Such terms as in Eq.(12) at $\mathcal{O}(\psi^2)$ are discussed in the context of a higher derivative fermionic field theory (for example, see [14]), in which a (Weyl) ghost field is included. Higher order terms of ψ in $S_{\rm higher\ der}$, e.g., terms at $\mathcal{O}(\psi^4)$ can be obtained from the results in Eqs. from (4) to (8).

In the above consideration, the apparently pathological higher derivative terms (12) appear in the NL SUSY model which is described by $S_{\rm VA} + S_{\rm higher\ der}$. However, we also find that the above NL SUSY model, $S_{\rm VA} + S_{\rm higher\ der}$, is equivalent to the standard NL SUSY V-A model described only by $S_{\rm VA}$ of Eq.(11). In order to show this, here we explicitly construct the *NL SUSY invariant* relation which connects the two NL SUSY actions, $S_{\rm VA} + S_{\rm higher\ der}$ and $S_{\rm VA}$.

Indeed, let us denote the N-G fermion field of the standard V-A model as ψ', i.e., $S_{\rm VA} = S_{\rm VA}[\psi']$ which is invariant under the NL SUSY transfomation generated

by ζ,

$$\delta_Q \psi' = \frac{1}{\kappa}\zeta - i\kappa(\bar{\zeta}\gamma^a\psi')\partial_a\psi'. \tag{13}$$

The form of NL SUSY transformation law (13) is the same as Eq.(2). We also consider the relation between $S_{\rm VA}[\psi']$ and $(S_{\rm VA} + S_{\rm higher\ der})[\psi]$, starting from the following ansatz,

$$\psi' \ = \ \psi \ + \ i\kappa^{\frac{1}{2}} \ \partial\!\!\!/\psi + \ \mathcal{O}(\kappa^{5/2}), \tag{14}$$

in order to derive Eq.(12) in the NL SUSY model, $(S_{\rm VA}+S_{\rm higher\ der})[\psi]$. Then we easily obtain next higher order terms in Eq.(14) such that the NL SUSY transformation (13) is reproduced by Eq.(2); namely, we have

$$\psi' = \psi + i\kappa^{\frac{1}{2}}\partial\!\!\!/\psi + \kappa^{\frac{5}{2}}(\bar{\psi}\gamma^a\partial\!\!\!/\psi\partial_a\psi - \bar{\psi}\gamma^a\partial_b\psi\gamma^b\partial_a\psi) + i\kappa^3\bar{\psi}\gamma^a\partial\!\!\!/\psi\partial_a\partial\!\!\!/\psi + \mathcal{O}(\kappa^{9/2}). \tag{15}$$

Here we can further continue to obtain higher order terms in the SUSY invariant relation (15). When we substitute Eq.(15) into the standard V-A action $S_{\rm VA}[\psi']$, the $S_{\rm VA}[\psi']$ exactly leads to Eq.(12) (up to the total derivertive terms which have been omitted from Eq.(12)) in addition to the $S_{\rm VA}[\psi]$. We also expect that the $S_{\rm VA}[\psi']$ leads to higher order terms of ψ in $S_{\rm higher\ der}[\psi]$, e.g., terms at $\mathcal{O}(\psi^4)$.

We summarize the results as follows. Adopting the ansatz (3) with the higher (first-order) derivative term of the N-G fermion, we have investigated for $N = 1$ SUSY the relation between the scalar supermultiplet of linear SUSY and the NL SUSY model including apparently pathological higher derivative terms of the N-G fermion besides the V-A action. We have explicitly shown that the component fields of the $N = 1$ scalar supermultiplet are consistently expanded in terms of the N-G fermion in the NL SUSY invariant way as Eqs. from (4) to (8). The (NL SUSY invariant) higher derivative action of the N-G fermion, which apparently includes a (Weyl) ghost field, has been discussed at the leading order in Eq.(12). By using this relation and by constructing the NL SUSY invariant relation (15), we have also explicitly proved the equivalence between the standard NL SUSY V-A model and our NL SUSY model with the pathological higher derivatives as an example with respect to the universality of NL SUSY actions with the N-G fermion. In other words, the NL SUSY invariant condition may give more general field redefinitions, which reproduce the standard V-A model.

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